# FFT – $^1/_n$-octave analysis – wavelet

For most acoustic examinations, a simple sound level analysis is insufficient, as not only the overall sound pressure level, but also the frequency-dependent distribution of the level has a significant influence on the perception of a sound event. This Application Note presents several analysis types for frequency-dependent sound level determination: the analysis with a filter bank of $^1/_n$-octave filters, the Fast Fourier Transform and the Wavelet analysis. The functional principles of these analysis functions as well as their advantages and disadvantages are explained. In this regard, not only the frequency resolution, but – of course – also the time resolution of the respective analysis plays an important role. Therefore, the following describes especially those analysis functions that are displayed as a function of time.

Fast Fourier Transform

The Fourier analysis is based on a mathematical theorem introduced by J. B. Fourier. The statement of this theorem can be summarized as follows: any periodic signal form can be interpreted as a superposition of discrete, periodic sine and cosine oscillations with different frequencies and amplitudes. Practical implementations of this theorem can be found in the Discrete Fourier Transform (DFT) and the Fast Fourier Transform (FFT). The results of the DFT and FFT provide discrete frequency spectra of a sampled time domain signal. The FFT is a variant of the DFT requiring less computing resources.

Before the transform, the signal has to be sectioned (windowed) on the time axis, therefore the original signal is subdivided into several blocks with N samples each. This is the so-called windowing. In an averaged analysis, *FFT (Average)*, the results of the individual blocks are averaged. In the time-dependent analysis variants, *FFT vs. Time* and *FFT vs. RPM*, the results of the individual blocks are displayed successively in a spectrogram. An averaged analysis is suitable for stationary signals, whereas the time-dependent analysis is the method of choice for non-stationary signals. Unlike the $^1/_n$-octave level analysis, the FFT is an analysis with a constant bandwidth. Figure 1 schematically shows the distribution of the frequency nodes of the FFT analysis on a linear and on a logarithmic frequency scale. The frequency nodes on the linear frequency scale are equidistantly distributed, whereas on the logarithmic frequency scale they are located closer to each other with higher frequencies. The frequency resolution ∆f is constant across the entire frequency range, it depends on the sampling rate and on the block length chosen for the analysis: $\Delta f = \frac{sampling\ rate}{block\ length}$
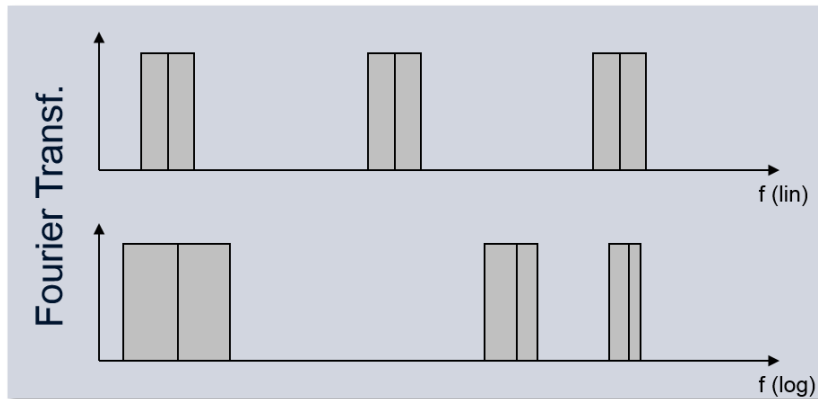
Figure 1: Schematic representation of the frequency nodes of the FFT analysis on a linear and a logarithmic frequency scale

Due to of the constant distribution of the nodes, averaged FFT analysis of pink noise shows a descending curve, while the analysis of white noise shows a frequency-independent curve (see figure 2).
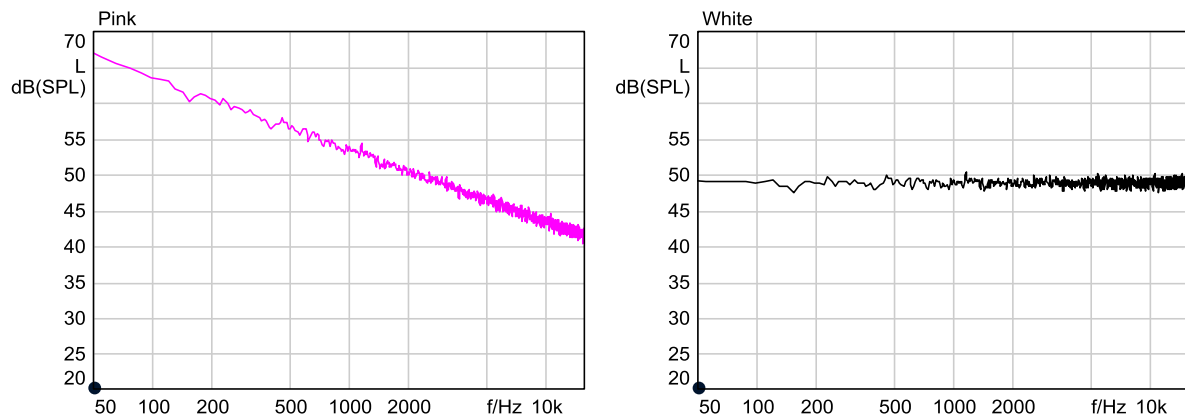
Figure 2: FFT analysis of pink noise (left) and white noise (right)

$1/_n$-octave analysis

In the $1/_n$-octave analysis, the signal to be analyzed is split into partial signals by a digital filter bank before the sound level is determined. The filter bank consists of several filters connected in parallel, each with a bandwidth of $1/_n$-octave. An octave filter is a filter whose upper cutoff frequency is twice the lower cutoff frequency, whereas $1/_3$-octave filters further subdivide each octave band into three parts and so on. This means that octave filters or $1/_n$-octave filters do not have a constant absolute bandwidth, but a constant relative bandwidth, i.e., the frequency bands are equidistant on a logarithmic frequency scale. Displayed on a linear frequency scale, the filter bandwidth increases logarithmically (see figure 3).
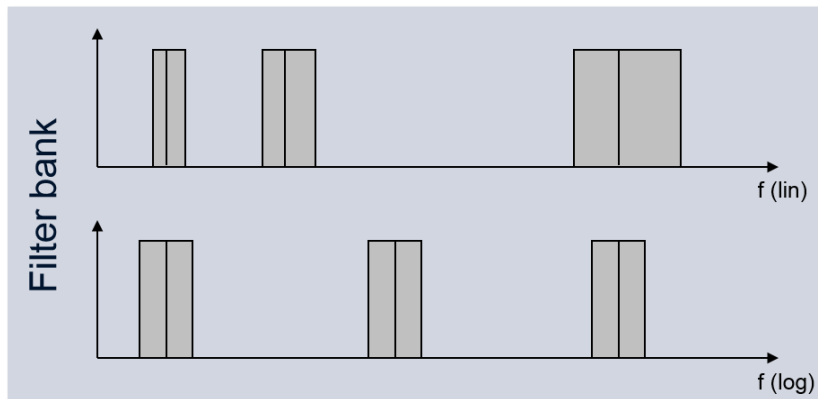

Figure 3: Schematic representation of the filter bandwidth on a linear and a logarithmic frequency scale

An octave filter with a center frequency of 63 Hz has a bandwidth of 44 Hz, whereas an octave filter with a center frequency of 16000 Hz has a bandwidth of 11360 Hz. A $1/_3$-octave filter with the center frequency of 63 Hz has a bandwidth of only 14.5 Hz and a $1/_3$-octave filter with a center frequency of 16000 Hz has a bandwidth of 3650 Hz. Towards higher frequencies, the bandwidth of these filters increases more and more.

The effects the filters broadening towards higher frequencies have on the analysis results are described below by means of pink and white noise. White noise has a frequency-independent spectral power density, i.e., the signal level is constant across the entire frequency range. When a white noise is analyzed with a filter bank of $1/_n$-octave filters, the diagram shows a rising curve, because the filter bandwidth and thus the amount of energy per filter band increases towards higher frequencies (see figure 4, right). When the same filter bank is used to examine a pink noise signal whose amplitude drops by 3 dB per octave towards higher frequencies, the power is approximately the same for each filter band. The power drop of the pink noise towards higher frequencies is therefore compensated by the increasing filter bandwidth. The $1/_n$-octave analysis of pink noise therefore shows a frequency-independent curve (see figure 4, left).
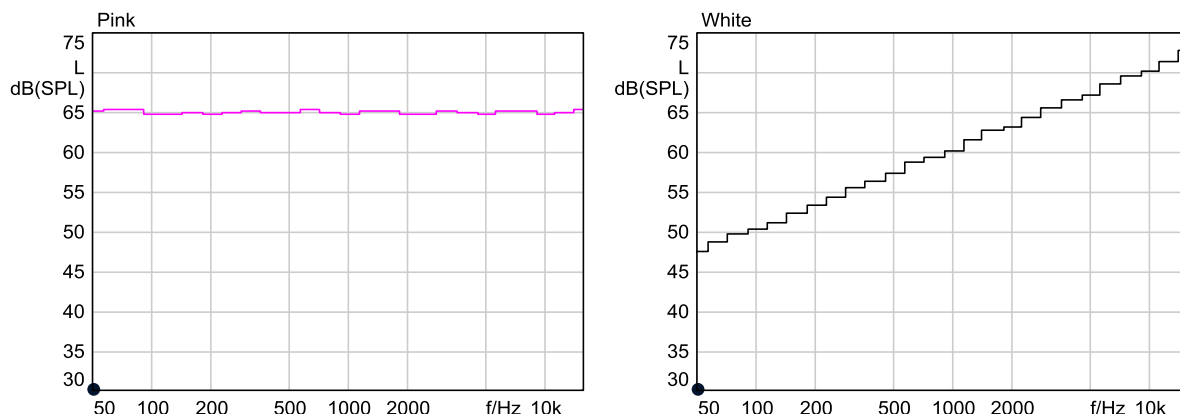

Figure 4: $1/_3$-octave band analysis of pink noise (left) and white noise (right)

As the result of a $^1/_n$-octave analysis, either an averaged spectrum or a time- or RPM-dependent spectrogram can be calculated. For this purpose, ArtemiS SUITE provides the analyses *$^1/_n$ Octave Spectrum*, *$^1/_n$ Octave Spectrum (Peak Hold)*, *$^1/_n$ Octave Spectrum vs. RPM* and *$^1/_n$ Octave Spectrum vs. Time*, each in two versions with the addition *FFT* or *Filter*[1], respectively. In the FFT-based analysis, the partial bands are calculated by adding the corresponding spectral bands from an FFT spectrum, whereas a filter-based analysis actually determines the partial bands by means of digital filtering. Unlike the continuous processing of the signal with the *Filter* method, the *FFT* method processes the signal block-wise, i.e., the signal is subdivided into blocks prior to the analysis. Since the *Filter* method does not require windowing of the signal and thus the thereby resulting artifacts can be avoided, it is preferable to the *FFT* method.

Wavelet

In the Wavelet analysis[2], the sound signal is examined using small wave packets called *Wavelets*. For this purpose, ArtemiS SUITE uses the impulse responses of different bandpass filters as Wavelet analysis functions.
Unlike the FFT with its constant analysis bandwidth, the Wavelet analysis (just like the $^1/_n$-octave analysis) has a frequency resolution with a constant relative bandwidth. Figure 5 illustrates the difference between the frequency and time resolution of the FFT and Wavelet analysis.
The x-axis in the drawing represents time, the y-axis represents frequency. A narrow, high box stands for a high time resolution and a low frequency resolution, whereas a flat, wide box symbolizes a good frequency resolution and a low time resolution. Due to the time-frequency blurring, the area of each of these "time-frequency" boxes in the figure is the same.
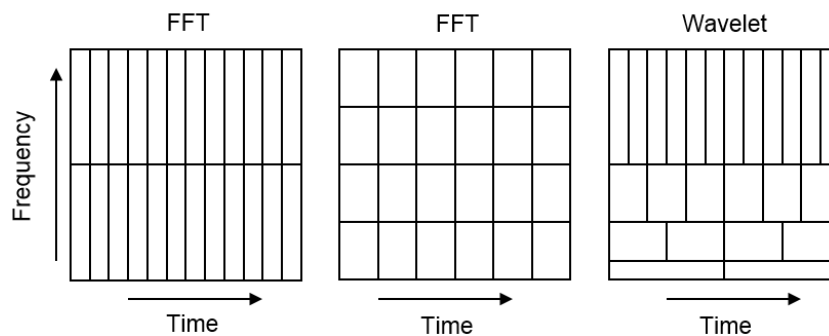
Figure 5:   Frequency and time resolution of the Wavelet analysis compared to the FFT

The left and center diagrams show the resolution of an FFT analysis with different window lengths. The left diagram shows the resolution of an FFT analysis with a small window length (i.e., a high time resolution), resulting in a low frequency resolution. The FFT in the center diagram has a larger window length, which reduces the time resolution, but improves the frequency resolution. Both the time resolution and the frequency resolution of the FFT are constant across the entire frequency range. This is not the case with the Wavelet analysis, as shown in the right diagram of figure 5. At low frequencies, the Wavelet analysis delivers a high frequency resolution combined with a low time resolution. Towards higher frequencies, the frequency resolution gets worse, but the time resolution improves significantly. The area of the boxes always remains constant.
The resolution of the Wavelet analysis is a good approximation of the analysis that takes place in the human ear.

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[1] These analyses can only be selected if the ArtemiS SUITE license used includes ASM 14, otherwise only the FFT-based analyses are available.
[2] This analysis can only be selected if the ArtemiS SUITE license used includes ASM 17.

Using the analyses in ArtemiS SUITE[3]

FFT analysis

Figure 6 shows the Properties window of the **FFT vs. Time** analysis which is described in detail below.
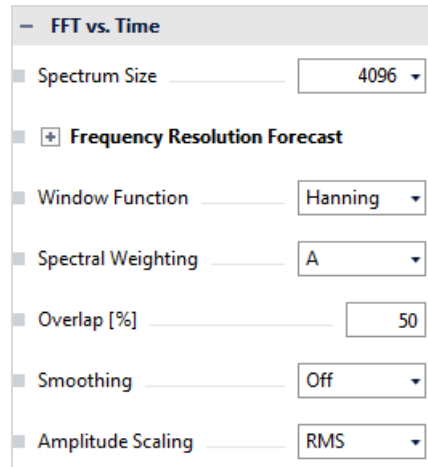
Figure 6:   Properties window of the **FFT vs. Time** analysis

In the selection box **Spectrum Size**, the block length for the analysis is selected. A large block length results in a high frequency resolution of the FFT analysis, whereas a small block length leads to a higher time resolution. Due to the temporal windowing, the FFT analysis is subject to a time/frequency blurring: high frequency resolution always results in low time resolution and vice versa. However, the possibility to specify the window size allows you to choose which of the two aspects is more important for your application. At a sampling rate of 44100 Hz, a block length of 4096 samples results in a time resolution of 0.093 seconds. The frequency resolution in this case is 10.77 Hz. By increasing or decreasing the block length, the frequency resolution or the time resolution is improved respectively. By maximizing the **Frequency Resolution Forecast** view, you can display the frequency resolution for the currently selected DFT length for different sampling rates.

In the next selection box, you select the desired **Window Function**. The window function allows a time weighting of the individual block sections in order to reduce the so-called leakage effect. As already described, the signal must be cut into a number of blocks for the FFT analysis. In the analysis of these blocks, a periodic continuation of the signal is assumed. This can lead to points of discontinuity at the borders of the signal section if no integer multiple of the period is contained in the block. These discontinuities result in frequencies in the spectrum that do not exist in the original signal. This "leakage" of signal energy into neighbor frequencies of the original frequency is what gives the leakage effect its name. By means of suitable windowing with window functions that go to zero at their borders, this effect can be reduced. Since the selection of the window function affects the analysis result, a window function suitable for the respective application must be selected. For many applications, the Hanning window is a good choice, as it greatly reduces the leakage effect. Other window functions are optimized for specific applications. For example, the Kaiser-Bessel window has a very good frequency resolution and should be used if separate tonal components with very different levels are to be separated from each other.

To compensate for the time weighting of the window functions, an overlapping of the windows can be specified. In the field **Overlap [%]**, the desired overlapping can be entered in percent. For the Hanning window, overlappings of 50 % or 66.67 % are often used. An overlapping of 50 % results in a windowing compensation with accurate amplitudes, whereas an overlapping of 66.67 % keeps the signal power constant.

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[3] The descriptions in this Application Note are based on version 9.2 of ArtemiS SUITE. The general procedure also applies to other versions. However, there may be differences in the scope of functions and in the user interface.

In the selection box **Spectral Weighting**, you can activate or deactivate filter for a frequency-dependent weighting. With these filters, different frequency areas are weighted differently. The A weighting is often used for the analysis of airborne sound signals. This weighting takes the frequency-dependent sensitivity of the human ear into account, because the A filter approximately corresponds to the inverted hearing threshold level of a person with normal hearing capabilities.[4]

With the **Smoothing** function, the calculated FFT spectrum curve can be smoothed. Smoothing can be used if the spectral distribution of a signal is to be calculated, for example for the creation of a non-recursive filter (FIR filter) based on these results.

In the selection box **Amplitude Scaling**, two possibilities are available: **RMS** and **Peak**. In the first case, each FFT line displays the effective value of the oscillation, in the second case the peak value is calculated, which is $\sqrt{2}$ times higher than the RMS value (for sinus oscillations).

$^1/_n$-octave analysis

Figure 3 shows the Properties window of the $^1/_n$ **Octave Spectrum vs. Time** analysis (left: **FFT** method, right: **Filter** method).
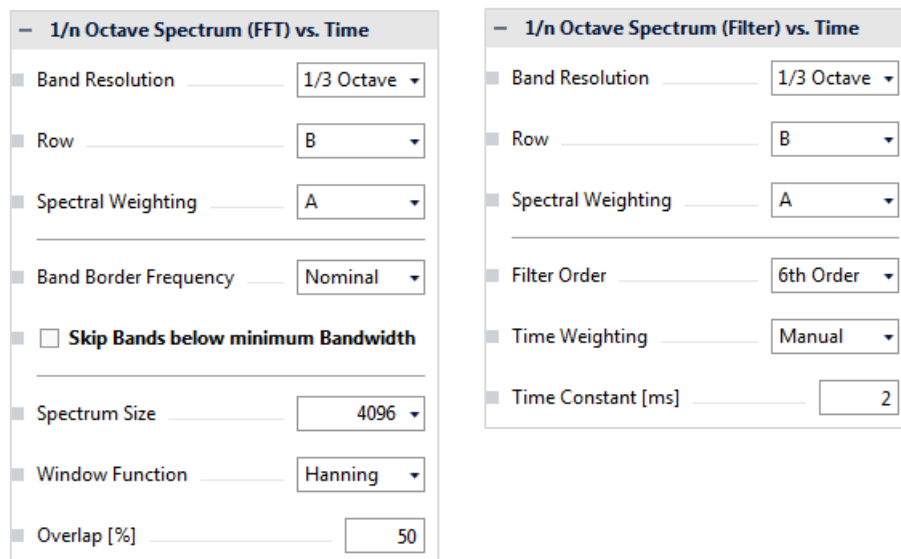
Figure 7:   Properties window of the $^1/_n$ **Octave Spectrum vs. Time** analysis (left: **FFT** based, right: **Filter** based)

In the first selection box of the Properties window, the **Band resolution** of the analysis is specified. Filters for octaves or for fractions thereof can be selected. To mimic human perception, $^1/_3$-octave filters are especially suited, because their bandwidth above 500 Hz approximately corresponds to the frequency groups used in psychoacoustics.

Besides the filter bandwidth, the frequency position of the filters can be adjusted as well. In the **Row** selection box, one of two filter rows can be selected, which are offset from each other by one half of the bandwidth. Row **B** is defined so that the 1 kHz mark corresponds to the center frequency of one filter. With the setting **A**, the 1 kHz mark corresponds to the cutoff frequency of one filter. Normally, row **B** is used.

In the selection box **Spectral Weighting**, you can activate a weighting of the analysis results with an A, B, C or D filter. With these filters, different frequency areas are weighted differently. The A weighting is often used for the analysis of airborne sound signals.

The other settings differ depending on the selected calculation method.

In the Properties window of the filter-based analysis you can configure the filter properties in the next section: in the **Filter Order** field filters of fourth and sixth order are available. The sixth order filters (Chebyshev filters with a ripple < 0.005 dB) comply with the ANSI standard S 1.11 and with IEC 61260.

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[4] The use and the effects of frequency weighting are described in the Application Note "Frequency Weighting of Airborne Sound Signals".

The $^{1}/_{3}$-octave filters of fourth order comply with the withdrawn DIN 45652 and are only available for compatibility reasons. Basically, the use of sixth-order filters is recommended unless you need to use fourth-order filters for comparability with previous analysis results. Furthermore, a **Time Weighting** setting can be chosen. This function smoothes the level curves by means of exponential integration. Available options are **Slow**, **Fast**, **Impulse** and **Manual**. The **Slow** integration period is 1 second and the **Fast** period is 125 ms. In the **Impulse** setting, a peak detector with an integration period of 35 ms and a decay time of 1500 ms is activated. If **Manual** is selected, the integration period can be specified by the user in the field **Time Constant [ms]**. The longer the selected integration period, the more the level curve is smoothed. To resemble the processing time of human hearing, an integration period of 2 ms should be used.

The other settings for the FFT-based analysis are described in the following. For the parameter **Band Border Frequency**, three options are available: **Nominal**, **Octave** and **Decade**. Depending on the setting, the limits of the frequency bands are determined differently, so the analysis results can differ slightly. With the **Nominal** setting, the band limits are taken from standardized tables. With **Octave**, the limits are calculated with the formula f = 1kHz · $2^{n/M}$, whereas the formula for **Decade** is f = 1kHz · $10^{n/M}$. M stands for the fraction of the octave or decade and n is a whole-numbered index.

The function **Skip Bands below minimum Bandwidth** allows you to enter a minimum bandwidth depending on the frequency resolution of the FFT. All frequency bands below this minimum bandwidth are not displayed. This function can be used to prevent frequency bands from being displayed for which no FFT node is available yet.

The last three parameters specify the windowing function used for calculating the FFT. They include the **Spectrum Size**, the **Window Function** and the **Overlap[%]**. These parameters have already been explained in the previous section.

Wavelet analysis

Figure 8 shows the Properties window of the Wavelet analysis in ArtemiS SUITE. Just like for the other analysis types, the first selection box allows a **Spectral Weighting** to be specified, which weights the calculated spectrum using an A, B, C or D filter.

Figure 8: Properties window of the **Wavelet** analysis

The next section contains parameters for the bandpass filter whose impulse response is to be used as the Wavelet analysis function. These parameters include the **Filter Type**, the **Filter Order** and the **Filter Quality**. Thus the rate of rise of the filter edges and the filter bandwidth are specified. A steep, narrow filter means a high frequency-resolution, but also a low time resolution (and vice versa). If the filter type selected is **Chebyshev**, the **Ripple** parameter for the frequency range of the filter must be specified in addition. In a Chebyshev filter, ripple affects the filter slope. At a constant filter order, the filter slope becomes steeper the more ripple is accepted.

By specifying the **Frequency Range** to be analyzed, you can limit the examination to the interesting area. In the last selection box of the Properties window, the required **Resolution** can be specified. If the

setting **High** is selected, 128 bandpass filters are used for the analysis of the specified frequency range, 64 filters are used for the **Medium** resolution and 32 filters for the **Low** resolution. A higher number of bandpass filters not only causes a better resolution, but also a longer computing time.

Just like the $^1/_n$-octave analysis, the Wavelet analysis in ArtemiS SUITE performs a time weighting by means of an exponential integration. However, the Wavelet analysis does not use a fixed, but a frequency-dependent time constant T defined as follows: $T = \dfrac{1}{center\ frequency\ of\ the\ band-pass\ filter}$

Comparison of the analysis methods

In the following, two sounds from the automotive area are to be analyzed: one is the sound of a car door being closed, the other is an engine sound with a clearly noticeable high-frequency whistling of the turbocharger. The car door generates a very short, broadband noise, whereas the engine noise contains a very distinct tonal component. For a meaningful analysis of these sounds, it is not only necessary to choose the right analysis type, but also selecting the correct analysis parameters. It must be considered that the correct analysis parameters not only depend on the type of sound, but also on the specific aspect of the sound that is to be examined in detail. For the analysis of the door closing sound, this means that even though the sound is very short, a large block length may be required for the FFT analysis if the individual frequencies of the sound are to be examined.

The figures below illustrate the difference between the analysis types and the effect of changing the parameter settings. Figure 9 shows analysis results of the door closing sound. The two diagrams on the left side show a $^1/_n$-octave analysis, the center diagrams show an FFT analysis, and the right diagrams show a Wavelet analysis. The difference between figure 9 and 10 is one respective parameter that has been changed for each of the calculations. The following analysis parameters were used:

$^1/_n$ Octave Spectrum vs. Time:

Band Resolution:	$^1/_3$ Octave (top) / $^1/_{12}$ Octave (bottom)
Spectral Weighting:	None
Method:	Filter
Filter Order:	6th Order
Time Weighting:	Manual
Manual (ms):	2

FFT vs. time:

|	
Spectrum Size:	512 (top) / 4096 (bottom)
Window Function:	Hanning
Spectral Weighting:	None
Overlap (%):	75
Smooth:	Off

Wavelet:

Spectral Weighting:	None
Filter Type:	Chebyshev 0.5 dB
Filter Order:	6th Order
Frequency Range:	20 to 16000 Hz
Filter Quality:	10
Resolution:	Medium (top) / High (bottom)

The comparison of the top diagrams shows that the frequency resolution of the different analysis types is different. In the FFT analysis, the frequency resolution is constant across the entire frequency range. However, since the frequency axis is shown logarithmically, the frequency resolution of the FFT analysis appears to be worse at low frequencies than at high frequencies. The $^1/_3$-octave level analysis (top left), just like the Wavelet analysis (top right) has a frequency resolution that is constant on the logarithmic frequency axis. Furthermore, the comparison shows that the results of those analysis types based on digital filtering ($^1/_3$-octave level and Wavelet analysis) have a slight delay at low frequencies. This is caused by the tuning process of the digital filters, which takes longer for low-frequency filters. The FFT

analysis does not involve such a tuning, therefore in its diagram the display of the low frequencies is not shifted.
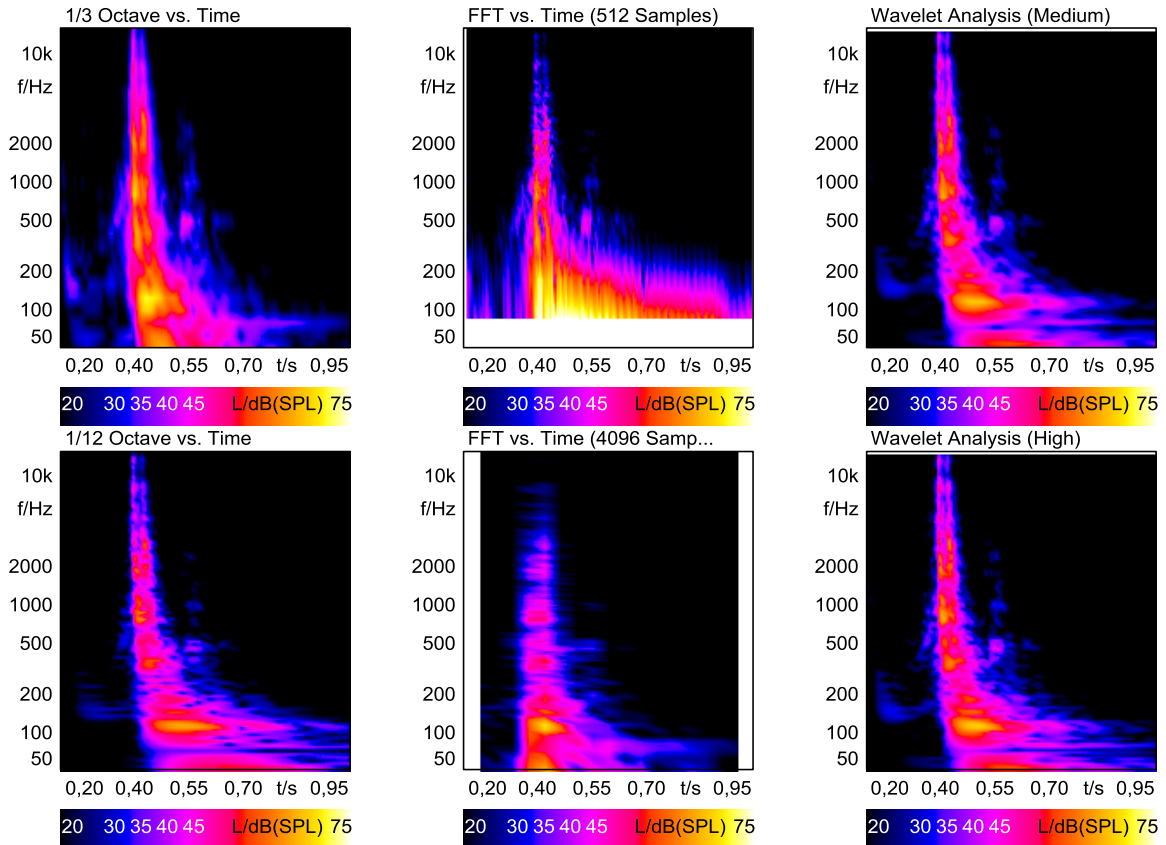
Figure 9: Analysis result of a door closing sound; left diagrams: *1/n Octave Spectrum vs. Time* (top: *1/3 octave*, bottom *1/12 octave*), center diagrams: *FFT vs. Time* (block length, top: *512*, bottom: *4096*), right diagrams: *Wavelet analysis* (resolution, top: *Medium*, bottom: *High*).

The difference between the top and bottom diagrams in figure 9 is the higher frequency resolution of the lower diagrams. The improvement of the frequency resolution, however, also has negative effects. In the $^1/_n$-octave analysis (left diagrams), the increasing of the resolution from $^1/_3$-octaves to $^1/_{12}$-octaves leads to a stronger temporal smearing of the analysis result at low frequencies. In the FFT analysis (center diagrams), the comparison shows that the better frequency resolution (which is best visible at low frequencies) leads to a very limited time resolution. In the Wavelet analysis, the effect of the changed analysis parameters is only marginal.

Altogether, the Wavelet analysis represents the frequency-time progression of the door closing sound very well. However, the other two analysis types reveal important details as well if the strengths and weaknesses of the respective analysis type are taken into account. Which analysis is eventually used depends, of course, also on the user's personal preferences and practice.

One advantage of the FFT analysis not mentioned yet is the much shorter calculation time compared to the Wavelet analysis. If quick analysis results of a large amount of data are needed, this would be a job for the FFT method.

Figure 10 shows the analysis results of an engine sound containing a tonal whistling component. For the analysis, the same parameters were used as for the door closing sound in figure 9, bottom. The comparison between figure 10 shows that for the analysis of this high-frequency tonal component, the FFT analysis with a block length of 4096 samples is especially well suited.

The whistling of the turbocharger between 4 and 5 kHz can be seen very clearly in this analysis. In the two other analysis methods, the frequency resolution at high frequencies is not sufficient to resolve the tonal component.
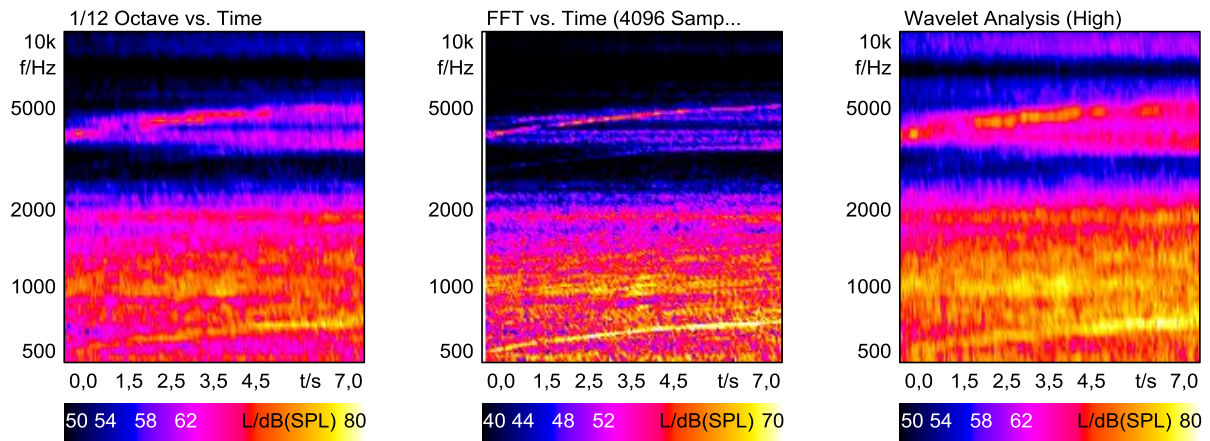
Figure 10: Analysis result of an engine sound; left diagram: *1/12 Octave Spectrum vs. Time*, center diagram: *FFT vs. Time* (block length: *4096*), right diagram: *Wavelet analysis* (resolution: *High*).

Do you have any questions or comments?
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