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Making Modal Analysis Easy and More Reliable – Reference Points Identification by Experimental Prestudy

Tim Kamper, Denis Beljan, Haiko Brücher, and Matthias Wegerhoff Head acoustics GmbH

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Abstract

Though modal analysis is a common tool to evaluate the dynamic properties of a structure, there are still many individual decisions to be made during the process which are often based on experience and make it difficult for occasional users to gain reliable and correct results. One of those experience-based choices is the correct number and placement of reference points. This decision is especially important, because it must be made right in the beginning of the process and a wrong choice is only noticeable by chance in the very end of the process. Picking the wrong reference points could result in incomplete modal analysis outcomes, as it might make certain modes undetectable, compounded by the user's lack of awareness about these missing modes.

In the paper an innovative approach will be presented to choose the minimal number of mandatory reference points and their placement. While other approaches use results of numerical simulations or rely on a visual evaluation of measurement data by the user, the presented approach is based on a few simple measurements and works automatically without any further user-interaction. In addition to traditional methods such as the Least-Squares Complex Frequency-domain (LSCF) estimator the presented approach takes advantage of a Neural Network to make user-interaction redundant.

The advantage of the presented approach will be shown based on the example of a real structure under test.

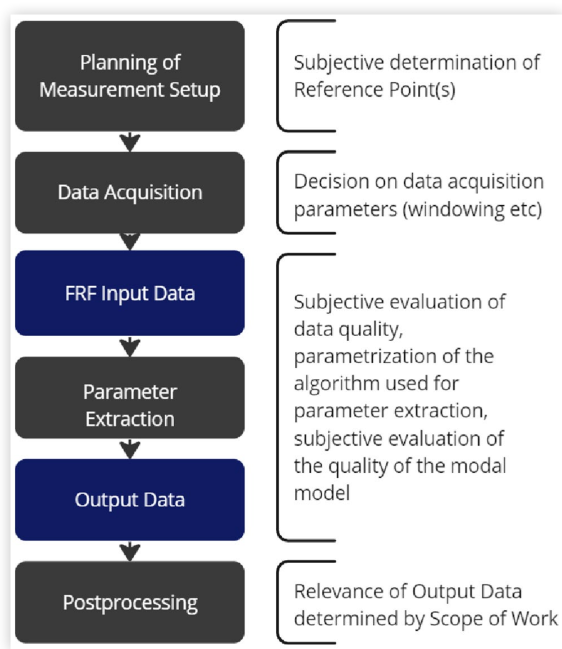
Introduction

Modal analysis has a long history and has become a standard tool across various industrial sectors today. It assists troubleshooters, developers, and simulation engineers in the search for the dynamic properties of the object under investigation. Until today, experimental modal analysis is primarily a tool used by experts though.

Throughout the entire workflow (as depicted Figure 1), it is necessary to make subjective decisions based on experience, thus influencing the result. Moreover, the uncertainty of the decisions and the respective results increases with the complexity of the test object. In the case of cantilever beams for instance, one can visualize which modes are to be expected and what to do to capture them, but with more complex components, there is less predictability regarding the expected outcome and at the same time it is more difficult to make experience-based decisions.

For every workflow-step depicted in figure 1 research is going on to ensure better results or tackle specific structural dynamic problems. In addition, great progress has been made in the past through the development of more advanced algorithms for parameter extraction and

FIGURE 1 Workflow of the experimental modal analysis (dark grey box: task, blue box: data)



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the creation of additional indicators for an in-depth analysis of the test object.

This paper is more about the simplification of the workflow instead of more advanced algorithms to analyze very specific structural dynamic problems. The goal of this paper is to contribute to more reliable results in modal analysis, regardless of the user's individual experience. It focusses on the first workflow step which is according to Figure 1 the planning of the measurement setup. Thus the following state of the art is restricted to this topic as well.

The planning of the measurement setup involves determining the total number and positions of roving DOFs (degrees of freedom), as well as the definition of one or more reference DOFs. The paper focusses on the definition of the reference DOFs.

If the DOFs are not correctly determined, all subsequent process steps can be influenced and may potentially yield an inaccurate representation of the vibration behavior of the investigated system [1].

This will be explained through examples of a free-free vibrating beam and an all-around fixed clamped plate (Figure 2). The provided examples serve to illustrate that the choice of the reference DOF (green/red circles) significantly contributes to the results.

In the left part of Figure 2, the first four bending modes are depicted. Placing the reference DOF at the midpoint of the beam results in the reference DOF coinciding with the nodal point for every other mode. This leads to the inability to determine the even modes, as there is no displacement at that particular point. In this extreme case, 50% of the existing modes would remain unextracted.

In the right part of Figure 2, the first six modes of an all-around fixed clamped plate are shown. If the reference DOF is positioned in the middle, only two of the first six modes can be extracted because the reference DOF, coincides with a nodal point for all the other modes.

For symmetric components, the scenario is straightforward. However, even a slight change in boundary conditions, such as altering the clamping conditions, makes the estimation increasingly complex and no longer easily predictable.

This leads to the outcome that, at the end of an experimental modal analysis, only a portion of the modes are identified, and the measurements may need to be repeated with a different setup, assuming the missing modes are even recognized as such.

After the introduction of the challenge of defining the reference DOFs, the user has several options to approach this problem:

Option 1:

If prior knowledge of the object is available, for instance, if it is a modified test object with a system behavior not significantly different from its predecessor, the already defined reference DOFs can be adopted and adjusted as needed.

Option 2:

The user has access to validated or not validated numerical models or results. Even though the resonance frequencies might still exhibit significant differences from reality, the mode shapes in the frequency range of interest can provide insight into the required number and location of roving and reference DOFs. In the past, different publications were issued on the topic of reference DOF selection based on numerical results [2, 3].

Option 3:

If neither of the first two options is feasible, the user must decide on reference DOFs solely based on their individual experience. For simple structures, an estimation can be made by the visual inspection of driving point measurements (DPMs) at different locations across the object. The DPM is a special type of frequency response function (FRF) where the excitation force and the resulting acceleration are measured at the same point or at least very close to each other.

The DPM can be compared and roughly evaluated using peak-picking. However, this approach is limited in its applicability, especially when the structure has high damping and closely coupled modes. In this case, making a robust decision based solely on visual criteria becomes challenging, as it is shown in Figure 3.

Furthermore, when there is no numerical model available, a certain number of measurements is necessary to be sure not to miss a mode of the structure under test. Experience has shown that 10 to 15 measurements deliver a reliable result for most structures. Thus, the effort necessary for a visual inspection is increasing.

Moreover, if the modal content of the set of measurements is known one can hardly differentiate between closely spaced modes and scattering of poles due to

FIGURE 2 Example for the importance of a good choice of reference points

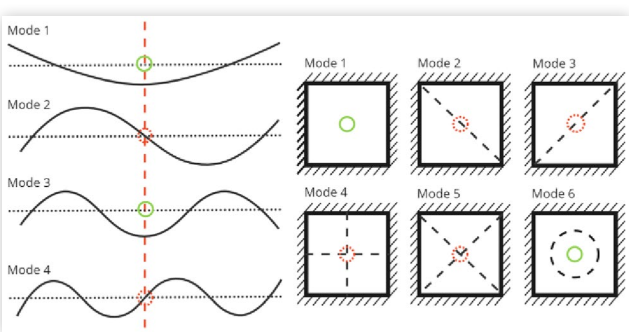
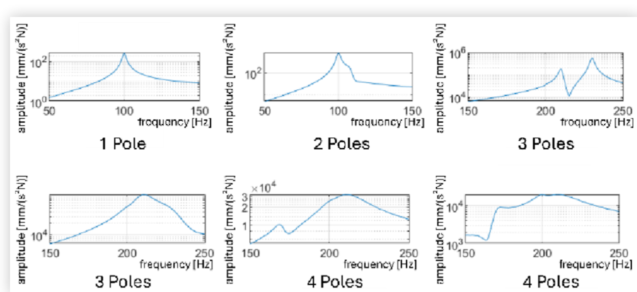


FIGURE 3 Examples for the variety of appearances of poles in FRF



measurement inaccuracies just based on visual inspection.

The method presented in this paper aims to standardize the determination of the optimal set of reference DOFs, making it more objective, and thereby increasing the likelihood of obtaining comparable and reproducible results, regardless of the individuals conducting the investigation. To make this feasible, different requirement to the approach where determined.

- The method should not require a CAD geometry or results of numerical simulation. It should solely rely on measured data.
- The method should work in an automated manner minimizing user interaction.
- No additional hardware to the hardware used for the modal analysis itself should be used.
- The additional effort for the user should be very limited and should take less time than 15 minutes.

The result should be a suggestion for the optimal set of reference DOFs based on the underlying measurement data and within the focused frequency range. If not all the modes present in the frequency range can be described by a single DOF, an additional reference DOF should be proposed. The goal is always to find the minimum number of required reference DOFs among all possible combinations.

The paper is structured as follows: The chapter "Description of the approach" starts with on overview over the necessary process steps of the approach and gives details about the implementation of the solution in the respective subsections. In the chapter "Example and results" an example structure is introduced and the results of the approach together with real measurement data are shown and validated. The paper concludes with a summary.

Description of the Approach

As stated above, the idea of the presented approach is to give a suggestion for the optimal set of reference DOFs solely based on a small experimental prestudy. Figure 4 shows an overview of the steps necessary for the approach, which will be described in more detail in the following subsections.

In a first step the frequency range of interest must be defined. After that initial step a DPM must be measured at one or more potential reference DOFs.

To give an example of how the input for the experimental prestudy could look like a small numerical model was set up. The numerical example has the advantage over test-based examples, that the modal content is known.

The numerical example consists of a plate of 10 mm thickness with the material properties of steel. The edges of the plate are 500 mm and 700 mm long and fixed to ground. As shown in Figure 5, the sample structure has 5 mode shapes in the range of 0 to 300 Hz. At three different locations on the plate, named DOF 1, DOF 2 and DOF 3, the DPM where simulated.

FIGURE 4 Flowchart of the approach

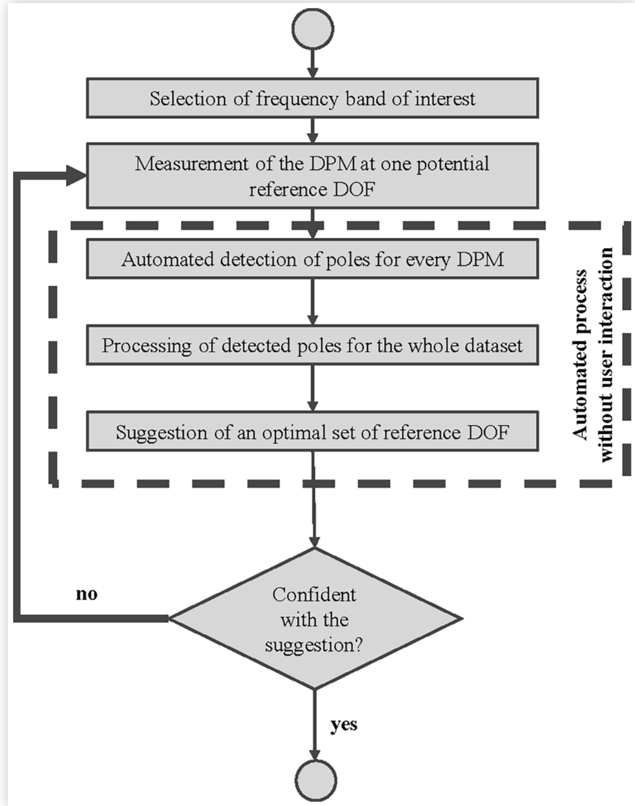


Figure 6 shows the DPMs at the 3 different DOFs marked in Figure 5. From visual inspection one could estimate that they are able to deliver different modal content if used as a reference DOF even though they all are located on the same structure with the same modal properties. While DOF 1 might be suitable to detect the mode at 76 Hz and 238 Hz, DOF 2 shows distinct peaks at 76 Hz, 138 Hz, 172 Hz and 229 Hz. For DOF 3 it is very difficult to inspect the modal content visually. This visual inspection of poles is an example of the option three discussed in the introduction.

While for this simple example with a simple structure and only three potential reference DOFs, the visual inspection of DPMs might be sufficient to pick a suitable set of reference DOFs, it gets more and more complicated for real structures and a realistic number of potential reference DOFs.

Figure 6 outlines a requirement for the experimental prestudy, in addition to those specified in the introduction. The objective is to progressively acquire a deeper understanding of the dynamic properties of the structure with each measurement. When only DOF 1 is provided as an

FIGURE 5 Numerical example and first mode shapes

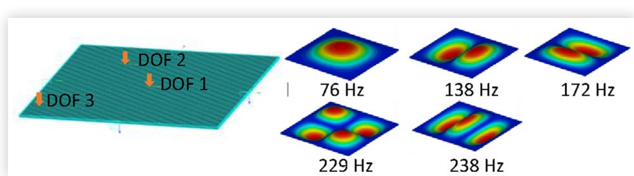
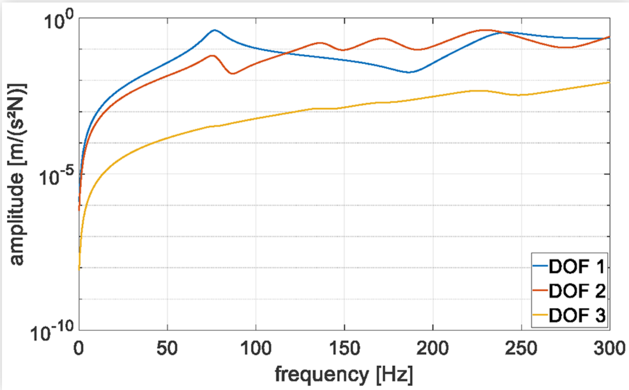


FIGURE 6 Example of numerically calculated DPMs for the points shown in figure 5



input to the prestudy, information about two modes becomes available. Introducing DOF 2 as an input further unveils additional modes of the structure. The evaluation of potential reference DOFs must be adapted according to this new, available information.

Therefore, the ongoing process is iterative and repeated with every measured DPM for all the DPMs measured so far (see Figure 4). This ensures that every additional information about the structure under test is considered in the evaluation.

To make the prestudy easy to use it is mandatory that the following evaluation of the DPM and the dynamic properties of the structure work automated and without user interaction. Especially because the evaluation steps must be repeated for every measurement as stated above.

The steps, that are part of the automated processing (see Figure 4), will be briefly introduced in the following paragraph and discussed in detail in the corresponding sections of the paper.

The automated processing of the measured DPMs begins with a pole detection to assess the modal information present in each of the respective DPM. In the next step the detected poles for the whole dataset are processed and compared to determine suitable points for exciting specific modes. Based on the results of this step, the optimal set of reference DOFs is suggested to the user. Since with every measurement more knowledge of the structure's dynamic properties is gained, the suggestion becomes more and more stable with a rising number of iterations. If the user is confident with the suggestion, the process ends. If the user is not confident with the suggestion, the process can be restarted by measuring a new DPM at a different potential reference DOF.

Automated Detection of Poles for Every DPM

To evaluate whether a specific DOF is suitable to detect a certain mode, the modal content of the DPM must be known.

Extraction of modal content from a single FRF or a set of FRFs is the central and thus a very common task in structural dynamics and modal analysis. To perform this task many methods were developed in the past. Since a DPM is only a special type of FRF, these methods can be applied to DPMs as well.

One prevalent and widely employed method to extract modal parameters from measured data is the Least Squares Complex Frequency method (LSCF) [4]. It is known to be robust and suitable for a wide range of frequencies and modal damping. For more details on the method please refer to the given reference. For this paper only the knowledge of the basic idea is important: The LSCF is an iterative approach that tries to fit a mathematical model of increasing order to the measured FRF. The optimal value for the highest requested order depends on the number of eigenfrequencies present in the dataset and must be chosen by the user. If the highest requested order is too low, not all eigenfrequencies can be detected, if it is chosen too high, mathematical poles arise, that do not represent an eigenfrequency of the structure.

There are other parameters necessary to extract poles with the LSCF such as:

- The number of iterations where a pole must be present to be regarded as a stable pole
- The frequency limits which describe how similar the frequency of the respective pole must be during this number of iterations
- The damping limit, which describes how similar the damping of the respective pole must be during this number of iterations.

However, the highest requested order is the most volatile parameter as it shows a strong dependency of the specific dataset, while the other parameters are seldom changed and mostly kept as default values.

Thus, to make the use of the LSCF easier and more reliable a neural network was developed that estimates the highest requested order based on the measured data without user interaction [5]. The following sections give more details on the neural network and how it contributes to the prestudy presented in this paper.

Automated Estimation of Highest Requested Order Based on a Neural Network

As stated above the highest requested order depends on the number of eigenfrequencies and corresponding poles present in a dataset or a single FRF. Therefore, the task that the neural network should perform is the estimation of the number of poles in a DPM. Neural networks have among other strengths a strong potential for feature detection and are widely used for many different applications. In the implementation and training of a neural network, careful consideration must be given to both the architecture of the network itself and the training process, including the data utilized for training. These topics are covered in the following subsections.

Network Architecture. The network architecture describes how a neural network is structured, e.g., which types of layers are used to build the network. To define the network architecture, at first the problem must be described in a way that a neural network is able to solve it. In this case the desired output is the number of poles present in the DPM. The potential input is the DPM itself. One possible solution is to build a classifier that sorts the DPM into different classes: e.g., zero poles, one pole, two poles and so on. This approach would lead to a high number of classes, which the network would have to differentiate. On the other hand, if the network is capable to detect the features of a pole, then the additional effort to know not only the total number but the position of the poles as well is neglectable. Thus, the desired output is a vector with as many entries as the input has frequency steps and for every frequency step the neural network should give a probability whether there is a pole at this specific frequency or not.

Keeping this aim of the neural network in mind, the next thing to think about are the features of a pole in a DPM. It is not possible to decide whether a DPM has a pole at a specific frequency step or not solely based on the respective frequency step alone. Every frequency step must be regarded in relation to the surrounding frequency steps. This task of evaluating a specific value based on its surroundings is commonly solved with the help of a U-Net based on convolutional layers.

Like it is depicted in Figure 7 a U-Net consists of two branches:

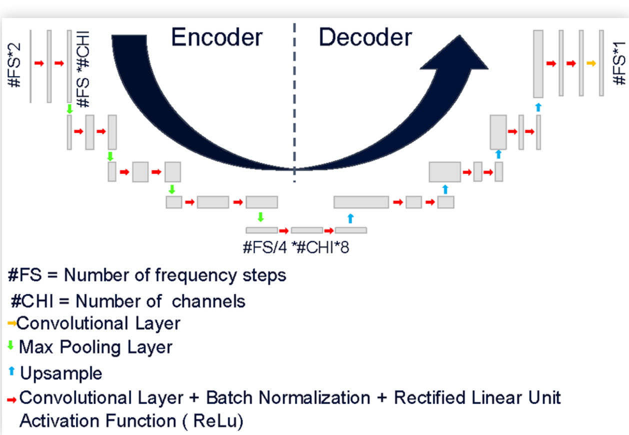
- The encoder is used to extract features from the input data. It reduces the spatial information and increases the feature information at the same time.

  The decoder is used to map the extracted features back onto the initial spatial representation (in this case the number of frequency steps).

Since the architecture of the U-Net is dependent on the size of the input, it is mandatory for the input to always be the same size. In our case each input consists of the imaginary and the real part of one DPM and it is the number of frequency steps which must be constant. To achieve this, the input is resampled to a fixed number of frequency steps before it is given to the neural network.

In addition to the general structure of a U-Net, consisting of encoder and decoder, Figure 7 lists the layers that are used to build the network.

FIGURE 7 Structure of the U-Net used for this prestudy [5]



An important layer for the U-Net to fulfill its task is the convolutional layer. In this case it is used in one dimension. It consists of several filters of a predefined width that slide along the data and calculate the weighted sum over the width of the filter. The weights are individual for each filter, and they are adapted during training. This way each filter of a convolutional layer is used to condense a feature of the input data to a corresponding feature map [6]. The batch normalization, which is commonly used in combination with a convolutional layer, helps to speed up convergence of the training by normalizing the input over the batch size [7]. Following the convolutional layer and the batch normalization, Figure 7 shows a piecewise linear function called rectified linear unit. It gives zero as an answer for negative inputs and for positive inputs it gives the input itself as an output. This adds non-linearity to the network and prevents the weights from getting stuck near zero or rising to very high values. Instead, the rectified linear unit increases the sparsity of the network forcing some weights to zero. Neural networks, which are sparse to a certain extent, learn faster and are less at risk to overfit. Overfitting describes an effect, where the network, rather than extracting features, remembers the data itself. This implies that it excels in performance on known data but struggles with the generalization to unknown data [8, 9]

The combination of convolutional layer, batch normalization and rectified linear unit is depicted as a red arrow in Figure 7.

The green arrows in Figure 7 represent max pooling layers. They are used to keep the focus on prominent features and neglect those, which are less prominent. This is done by a moving filter which calculates the maximum value of its input over its width. [10]

The blue arrows in Figure 7 indicate upsampling layers. These are used to map the extracted features back onto the initial spatial representation, in this case back onto the initially existent number of frequency steps.

Datasets for Training, Validation, and Test. To train a neural network, like the one designed and used in this study, a set of labeled data is needed. Labeled means in this case, that the expected output of the neural network comes with the data.

The dataset should be divided into three parts. The major part is used to train the network. This means, the input data is fed into the network and the output of the network is compared to the desired output. Based on the error between actual and desired output, the weights of the neural network's neurons are adapted, and the procedure is repeated.

Another part of the dataset is used do validate the network. Validation means the adaption of hyperparameters like batch size, learning rate etc. Furthermore, the validation set is used to check for overfitting between the training epochs.

The third part of the data used as a test set. The test set is a set of data, which is totally unknown to the network during training. It is used after training to check the network's performance on unknown data.

In addition to the fact that the expected output of the neural network for the specific DPM must be provided, there are more specifications that need to be fulfilled: the data must be similar enough to enable the neural network to converge but at the same time diverse enough to enable generalization to unknown data. Furthermore, a huge number of datasets is necessary. Since the neural network receives the DPM one by one, without context, there is no need for consistent sets of DPM though. Especially the demand for many datasets, for which the expected output of the neural network is known, is difficult to meet using real measurement data. In this case approximately 700000 DPM were needed. Therefore, it was decided to train the network on synthesized data. Synthesized DPM have several advantages:

- They are available in an unlimited number.
- The modal content and thus the expected output of the neural network are known.
- The dataset can be designed to meet every demand. If a specific set of parameters is hard to train for the network, more DPM with a similar set of parameters can be added.

To synthesize the DPM a simple formula was used, which is known form the curve fitting process [11]:

$$H_{pq}(\omega) = \sum_{r=1}^{N_m} \left(\frac{A_{pqr}}{j\omega - \lambda_r} + \frac{A_{pqr}^*}{j\omega - \lambda_r^*} \right) \tag{1}$$

Where,

$H_{pq}(\omega)$ = transfer-function from p to q in dependency of the angular frequency ω

λ = pole (λ^* = complex conjugate pole)

A = residue (A* = complex conjugate residue)

N_m = number of poles

The residue A can be calculated using the following equation:

$$A_{pqr} = \frac{1}{j2\omega_{dr} m_r} \psi_{pr} \psi_{qr} \tag{2}$$

Where,

ω_{dr} = damped angular eigenfrequency of the mode r

m_r = vibrational mass of the mode r

ψ_{qr} and ψ_{pr} = shape coefficients

The decision to use synthesized DPM based on the formulas (1) and (2) lead to the possibility to create DPM with randomized input. Among other parameters, the number and frequency value of eigenfrequencies were randomized, furthermore, the modal damping, the shape factors and the vibrational mass. To make the generalization to real measurements easier, different levels of noise were added to the synthesized DPM. It is important to notice that in this case it is not necessary to exactly mimic experimental data. Instead only the features of poles present in experimental data need to be present in the synthetical DPM as well.

Results of the Neural Network. Figure 8 shows the result of the neural network on the DPMs derived from the model introduced in Figure 5. Especially regarding DOF 3 the additional value compared to a visual inspection is proven. The neural network detects poles that can hardly be detected by visual inspection.

While Figure 8 shows the results of the neural network on numerical test data which was not used for training, Figure 9 shows the results of the neural network on data from real measurements of different structures, which was neither used for training. The DPM shown in Figure 9 where selected to show different mode densities, numbers of modes and damping ratios.

Figure 9 reveals, that the estimation of the neural network is not always of the same quality. Anyhow it is suitable to outperform the human user, keeping in mind the speed of the estimation compared to visual inspection. The neural network fulfills the necessary quality criteria because it is not used as a pole estimator on its own, but to parameterize the LSCF instead, like shown in the following section.

Combination of the Neural Network with the LSCF. To use the neural network in combination with the LSCF the approximate positions of the poles are neglected. Instead,

FIGURE 8 Result of the neural network on the numerically calculated DPMs for the points shown in figure 5

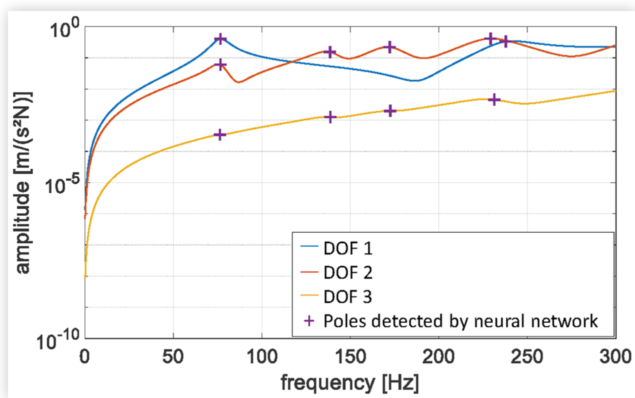


FIGURE 9 Results of the neural network on measured data taken from different structures

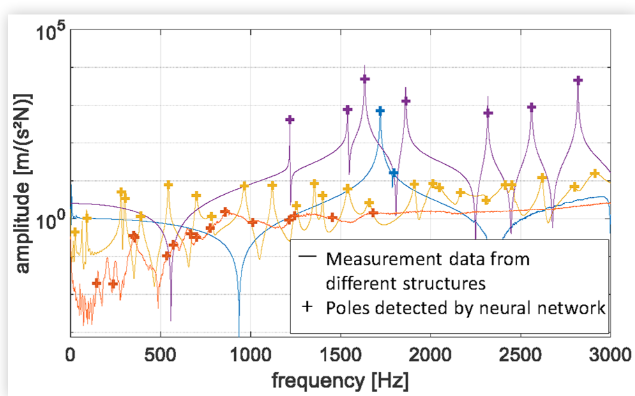
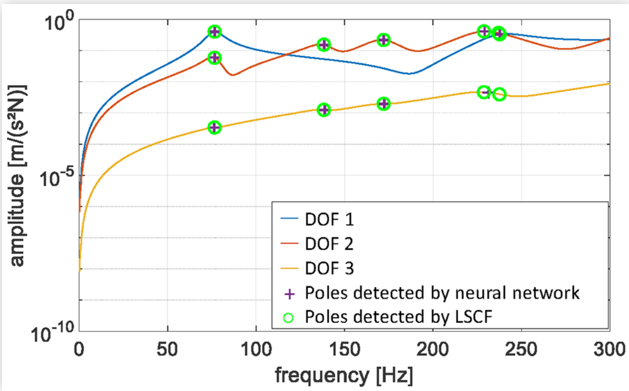


FIGURE 10 Result of the combination of neural network and LSCF on the numerically calculated DPMs for the points shown in figure 5



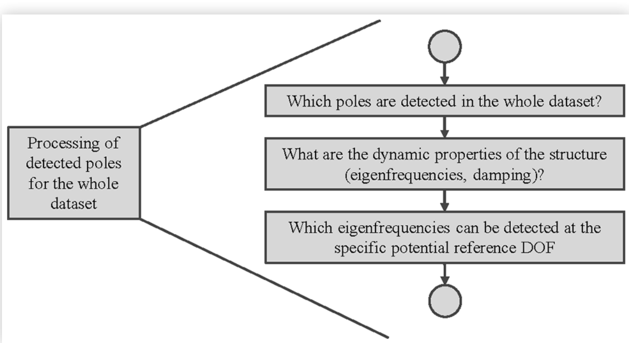
only the number of poles in every DPM is used and given to the LSCF as an estimation for the highest requested order. The results of this approach are shown in Figure 10.

In addition to the frequency of the poles which is more precise than the output of the neural network, the LSCF gives the modal damping of the respective poles. That is needed in the following steps as an additional input.

Processing of Detected Poles for the Whole Dataset

After for every measured DPM the poles are detected automatically, like it was described in the previous section, there are some steps to be performed to derive the dynamic properties of the structure from the detected poles. The different steps to process the detected poles are shown in Figure 11. The steps are performed iteratively every time a new measurement is added to the dataset for all measurements in the dataset. This is important, because the approach is based on continuous learning about the properties of the structure. With every new measurement there is the chance to learn more about the number and frequency values of the eigenfrequencies as well as the range of modal damping present in the structure or the maximal amplitude of the DPM. The poles detected in every DPM are classified based on all the

FIGURE 11 Flowchart processing of detected poles



information about the structure's dynamic properties available at this specific iteration of the prestudy.

In a first step all the poles from all measurements are collected in one big group. The poles in the group are sorted from lower to higher frequencies.

In a second step the poles detected in all DPM are grouped into subgroups representing the different eigenfrequencies of the structure. This step depends on the mode density. The accuracy of this step increases with every added measurement until all the eigenfrequencies of the structure in the specific frequency range of interest are known. Since the prestudy is based solely on measurements, there is no sharp stopping criteria indicating, that all eigenfrequencies are known. Instead, it is recommended to check whether the number of detected eigenfrequencies still changes after a few iterations. If it remains stable it can be concluded, that all eigenfrequencies where detected.

The third step is performed for every DPM separately. While the first two steps focus on the dynamic properties of the whole structure, the third step focuses on the properties of the specific DOF and how it contributes to the previously derived properties of the structure. To answer this question, the poles present in the specific DPM are compared to the eigenfrequencies of the whole structure derived in the previous step. Based on this evaluation a vector with as many entries as the number of eigenfrequencies is created. If the specific eigenfrequency is not present in the DPM, the respective value is zero. If it is present, the value is equal to the amplitude of the imaginary part of the DPM. The imaginary part of the DPM is taken as a measure of how good the respective eigenfrequency can be detected using the respective DOF as a reference point. The vectors of all the DPM sum up to the matrix M, which is used in the following step.

$$M = \begin{bmatrix} Im_{11} & \cdots & Im_{1m} \\ \vdots & \ddots & \vdots \\ Im_{D1} & \cdots & Im_{Dm} \end{bmatrix} \quad (3)$$

Where,
D = number of DOF
m = number of Eigenfrequencies
Im_{Dm} = normed amplitude of the imaginary part of the DPM at DOF D and for frequency m

Suggestion of an Optimal Set of Reference DOF

The result of the previous step is matrix M. The rows of matrix M represent the DOF and the eigenfrequencies are represented in the columns. The values range from zero to one as they represent the normed amplitude of the imaginary part of the DPM. This value is used to estimate, how good the specific mode can be excited or measured at the specific DOF. While one represents the highest amplitude in the dataset and thus an optimal detectability, a value of zero means, that the specific mode cannot be detected at the specific DOF.

In [2] an iterative algorithm is introduced which is used to find the optimal reference points based on data generated by a numerical modal analysis. The algorithm is designed to handle big amounts of potential reference points that can occur as a result of numerical modal analysis. But the algorithm can be used in a similar way for experimental data, generated using the experimental prestudy that is focus of this paper.

The algorithm accepts the previously mentioned matrix M as an input and in addition it needs a threshold value which defines the minimal requested amplitude for a mode to count as excitable. The threshold is set automatically depending on the damping present in the structure under test. The damping of the structure comes as an additional result of the previously conducted pole detection using the LSCF.

As output the algorithm gives the minimal number of requested reference DOF and gives a set of DOFs, which are optimal to use as reference DOFs. In the following the algorithm is split up into 7 steps.

1. In a first step, a temporary matrix (M_{temp}) is derived from M where all values below the threshold are set to zero, while the values above are set to one.
2. For every line (DOF) of M_{temp} all the values are summed up. The lines with a value of zero are erased because the corresponding DOFs do not excite any mode. The DOFs with the highest sum are then taken as potentially first reference DOFs.
3. For every potentially first reference DOF an own submatrix is derived, where the columns of the modes already detected by the first reference DOF are erased.
4. In every submatrix the values are summed up for every line and the lines with a sum of zero are erased again. The DOFs with the highest sum are added as a second reference DOF to the respective first reference DOF.
5. Steps 3 and 4 are repeated until there are no columns left in the matrix. This means that there are no modes present in the structure, that are not detected by one of the chosen reference points.
6. The minimal number of reference points needed is represented by the smallest group derived from the process explained above. All larger groups are deleted. The process results in several groups of reference points which are suitable to detect all modes present in the dataset. To find out, which one is the best, for every set a submatrix of matrix A is derived and the normed amplitudes for the DPM at the specific modes are considered. The set where the minimum of the normed amplitudes has the highest value is regarded to be the optimal set of reference DOFs for this iteration.
7. To find the global optimum the threshold is set to the previously derived minimum of the normed amplitudes and steps 1 to 7 are repeated until the minimal number of requested reference DOFs increases. The result of the previous iteration is then the optimal set of requested reference DOFs.

Example and Results

In the following section the results of the test based prestudy will be shown using the example of a torque support of an e-drive (see Figure 12). In the following section the results and the validation of the approach will be discussed in 3 steps.

To provide the necessary data to validate the approach the DPMs at 12 different DOF on the structure were measured using an impact hammer and an accelerometer. The measurements are performed in free-free conditions.

Figure 12 displays a 3 D model of the structure that was chosen as an example. The yellow dots depict the position of the chosen potential reference DOF at which the DPMs were measured. The direction was perpendicular to the structure's surface in every case. The positions are spread all over the geometry. To enable a comparability to and validation against the numerical prestudy presented in [2] the same structure was used under identical conditions. Furthermore, the DOF identified as the optimal reference DOF with the help of the numerical prestudy had to be in the set of potential reference DOF.

All the measured inertances where then evaluated with the approach presented in the sections above. The first result of this exemplary study is that for this structure one reference DOF is enough to excite all the modes between 0 Hz and 5000 Hz and the second result is, where this reference DOF should be located to gain optimal results of the modal analysis.

FIGURE 12 Example structure and chosen potential reference points

To make this result easier to interpret the matrix given in the upper part of Figure 13 can be derived. The rows of the matrix represent the measured DOF while the columns represent the eigenfrequencies of the structure derived from the measurements during the prestudy. It is important to know that these frequencies might differ a little from the eigenfrequencies identified during a full modal analysis, because the prestudy runs on a limited set of DPMs instead of a full matrix of FRF. Furthermore, in the case of the prestudy, the DPMs are fed one by one to the LSCF, whereas a full DPM matrix is fed to the LSCF during a full modal analysis.

In the matrix in Figure 13 a green entry represents the information that the specific eigenfrequency can be detected at the respective DOF. A red entry means that the specific DOF is not able to detect the respective eigenfrequency.

The DOF identified by the prestudy as the optimal reference DOF is marked with a blue box in Figure 13.

As a *first step of validation* the DPM for different potential reference DOF can be visually inspected. In the lower part of Figure 13 the DPM of the optimal reference DOF suggested by the tool (blue) is compared to the inertance measured at a bad reference DOF (orange) chosen based on the information provided by the matrix above. Like it was predicted by the matrix there are several modes present in the blue curve that will probably be more difficult to detect in the orange curve in the step of parameter extraction during a modal analysis. These are marked in red. This is why the optimal reference DOF suggested by the approach is superior and is suitable to ensure a complete modal model.

As a *second step of validation* the results of the prestudy presented in this paper are compared to the results of the numerical prestudy presented in [1]. While the test based prestudy relies on measurements and only DOFs that are measured can be evaluated regarding their suitability as refence DOF, the numerical prestudy relies on a finite element model of the structure. Thus, for every node on the surface of the model there is a rating available regarding the suitability as a reference DOF. This rating is visualized as a 3D Heatmap like it is displayed in Figure 14 on the left. The color scaling gives an impression on how good the reference DOF excites the modes. While nodes that are not suitable to detect all modes are displayed in black, the optimal reference DOF is displayed in white.

The optimal reference DOF suggested by the numerical prestudy is the point marked by a green circle in Figure 14. On the right of Figure 14 it can be seen that the reference DOF suggested by the test-based prestudy matches the result of the numerical prestudy.

As a *third step of validation* an experimental modal analysis is conducted using the reference DOF suggested by the test based prestudy. The measurements are conducted with the method of roving hammer and again the structure is measured in free-free condition. The extraction of modal parameters is done with the help of the LSCF. In parallel a numerical modal analysis of the structure was conducted using a FE-Model. The derived mode shapes of the experimental and of the numerical modal analysis are then compared using the modal assurance Criterion (MAC) [12].

The MAC is a measure of the correlation between two modal shapes. It can take values from zero to one, with zero meaning no correlation and one meaning identical mode shapes. Usually, MAC is shown as a matrix to compare two sets of mode shapes. In this case the columns represent the mode shapes derived from the experimental modal analysis and the rows represent the mode shapes derived from the numerical modal analysis.

The MAC matrix in Figure 15 reveals a good match between the results of the numerical and the experimental modal analysis in the frequency range of interest defined earlier. A perfect match would mean MAC values of one on the diagonal and zero everywhere else in the

FIGURE 13 Result of the prestudy

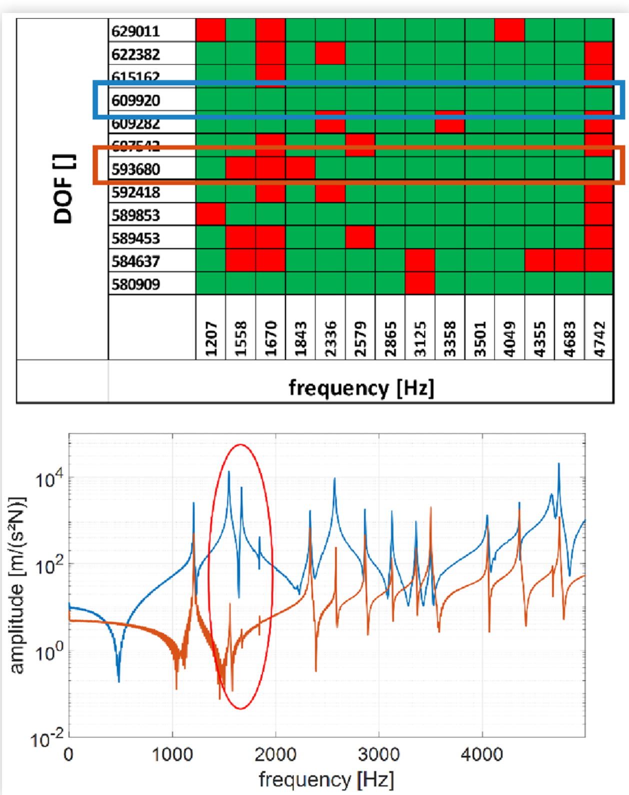


FIGURE 14 Comparison between numerical [2] and test-based prestudy

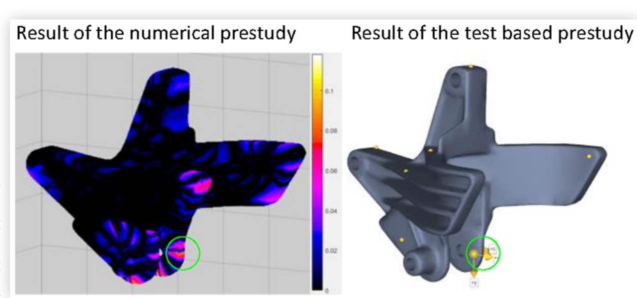
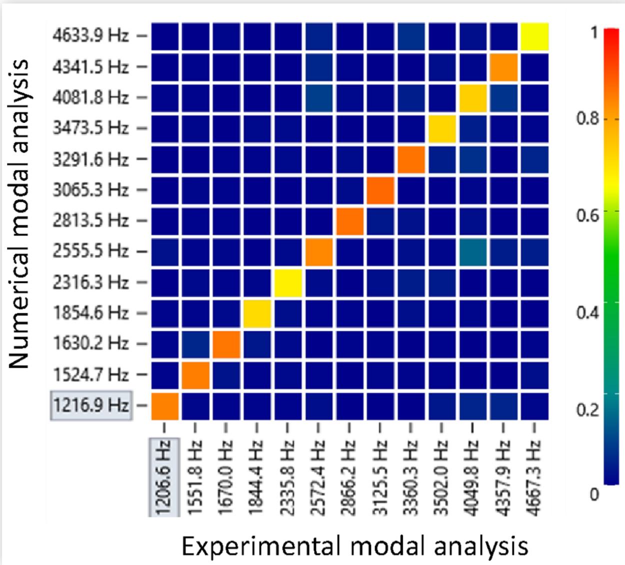


FIGURE 15 MAC matrix to compare the results of experimental and numerical modal analysis [2]



matrix. Small deviations in frequency and mode shape are caused by uncertainties in material parameters or geometry of the numerical model. Since this evaluation is not focused on the quality of the numerical model but on the completeness of the modal model derived from the experimental modal analysis using the suggested reference DOF, the slight deviations in frequency or mode shape do not diminish the excellent result. The reference DOF suggested by the test based prestudy ensures the completeness of the modal model.

Summary

In this paper, modal analysis is targeted as an important method for evaluating structural dynamics. Although it is frequently used, it is still a method that is mainly used by experts, as it requires many decisions that can only be made based on experience.

These experience-based decisions are necessary throughout the entire process, from planning, data acquisition and parameter extraction to post-processing. Often, wrong decisions at the beginning of the process can be recognized, if at all, only at the very end of the process. The incompleteness of a modal model is one example of the effect of a wrong decision in the initial planning step of the modal analysis. Incompleteness of the modal model means that not all modes present in the structure are captured in a particular frequency range and may happen due to a wrong selection of reference DOF(s). As the modal model becomes available only in the final step of the process, any errors can only be detected at the earliest in this last step. Moreover, if there is no reference to compare with, it is difficult to notice that one or more modes are missing, which means the error may not even be noticed at all.

This paper focuses on this decision on the selection of the optimal reference DOF, which must be made in the planning step. Choosing the wrong or too few reference DOFs can lead to an incomplete modal model. Choosing too many reference DOFs leads to a very high effort in instrumentation, measurement, and analysis.

The approach presented in this paper results in a proposal for the optimal set of reference DOFs, making the planning step of modal analysis simpler and more reliable. In this case, reliability means that the modal model derived from the measurements is complete within the frequency range of interest.

The approach presented consists of a small test based prestudy using the structure under test. No additional numerical model neither a CAD model is necessary. The approach is automated with the help of a neural network and works without user interaction. The only thing the user is requested to do is to measure some DPMs at the location of potential references DOFs. The evaluation of the potential reference DOF by the approach runs in parallel to the measurements and gives immediate feedback to the user.

The approach is introduced using data from an easy-to-understand numerical example. This has the advantage, that the numerical content of the structure is known beforehand and thus it is easy to evaluate the interim results of the approach.

Finally, the approach is validated using measurement data from a real use case. The transfer from the simple numerical example to the more complex, real structure of a torque support makes it possible to demonstrate the advantages of the approach. The test based prestudy suggests solely based on 12 DPMs at different potential reference DOFs, that one reference DOF is enough to detect all modes in the frequency range of interest. The suggestion is proven by means of an experimental modal analysis conducted using the respective DOF as a reference DOF.

Thus, another important step in context of experimental modal analysis towards reliable results, that are independent from the user's individual experience, is done.

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Contact Information

Dr.-Ing Tim Kamper
HEAD acoustics GmbH
Ebertstr. 30a
52134 Herzogenrath
tim.kamper@head-acoustics.de

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